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A relativistic signature in large-scale structure: Scale-dependent bias from single-field inflation

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In General Relativity, the constraint equation relating metric and density perturbations is inherently nonlinear, leading to an effective non-Gaussianity in the density field on large scales – even if the primordial metric perturbation is Gaussian. This imprints a relativistic signature in large-scale structure which is potentially observable, for example via a scale-dependent galaxy bias. The effect has been derived and then confirmed by independent calculations, using second-order perturbation theory. Recently, the physical reality of this relativistic effect has been disputed. The counter-argument is based on the claim that a very long wavelength curvature perturbation can be removed by a coordinate transformation. We argue that while this is true locally, the large-scale curvature cannot be removed by local coordinate transformations. The transformation itself contains the long-wavelength modes and thus includes the correlation. We show how the separate universe approach can be used to understand this correlation, confirming the results of perturbation theory.

I. INTRODUCTION

In Newtonian gravity, the Poisson equation is a linear relation between the gravitational potential and the matter overdensity. By contrast, in General Relativity (GR) this is replaced by a nonlinear relation, which introduces mode coupling between large and small scales. As a consequence, GR couples the process of halo and galaxy formation on small scales to long-wavelength perturbations of the geometry. One of the observational signals of this long-short mode coupling is a scale-dependent correction to the bias on large scales, as shown by [1–3]. The original result has been confirmed by a number of independent calculations [4–9].

The same scale-dependent bias can be produced by local-type primordial non-Gaussianity of the gravitational potential [10, 11]. The GR effect can thus be interpreted as an effective local non-Gaussianity on very large scales, in addition to any primordial non-Gaussianity. An observational detection of this bias would be a measurement of the sum of the two effects. In the case of a Gaussian primordial gravitational potential, there would still be scale-dependent bias due to GR nonlinearity.

Recently two papers have argued that a "separate universe" approach can be used to show that *no* scale-dependent bias arises from the GR corrections on large scales [12, 13]. The claim is that the nonlinear coupling between long-wavelength perturbations on a scale λ_L, and the small-scale variance, $\sigma_S^2 = \langle \delta_S^2 \rangle$, on a scale λ_S, vanishes under a local coordinate rescaling and hence is unobservable.

The separate universe approach [14, 15] has proved to be a powerful tool to understand the origin of large-scale structure, and primordial non-Gaussianity, from inflation. Accelerated expansion in the very early Universe stretches initial small-scale vacuum fluctuations up to scales much larger than the Hubble scale at the end of inflation. Spatial gradients for such long-wavelength modes become small relative to the local Hubble time, and for many scales of interest, the perturbed universe can be treated as a patchwork of "separate universes", each locally obeying the classical FRW evolution of an unperturbed universe.

The separate universe approach is particularly powerful for studying nonlinear perturbations on large scales [14, 16]. For adiabatic perturbations, each separate universe patch follows locally the same evolution as the unperturbed "background" cosmology. The only difference between separate patches is the local expansion, characterised by the comoving metric perturbation ζ. This is defined to be the local perturbation of the integrated expansion rate with respect to a background flat reference cosmology, $\delta N = N - \bar{N}$, where $N = \int dt\, \Theta/3$.

An important consequence of the uniqueness of the local evolution for adiabatic perturbations is that ζ is conserved on large scales where the separate universe approach is valid [15, 17, 18].

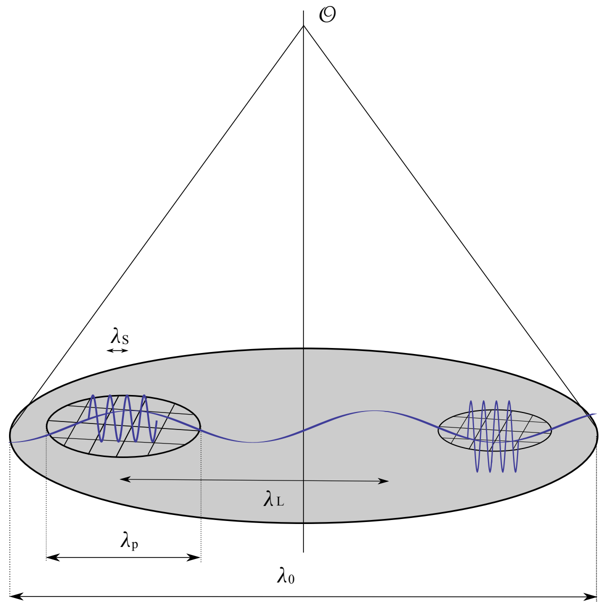

FIG. 1: Schematic of the various scales in (2).

In each patch, the comoving spatial line element is (see [13])

$$ds^2_{(3)} = e^{2\zeta}\delta_{ij}dx^i dx^j\,. \tag{1}$$

There is a global *background* which must be defined with respect to some scale λ_0, at least as large as all the other scales of interest, i.e., at least as large as our presently observable Universe. It is important to distinguish this from the scale of the separate universe patches, λ_P. This is large enough for each patch to be treated as locally homogeneous and isotropic, but patches must be stitched together to describe the long-wavelength perturbations on a scale $\lambda_L \gg \lambda_P$. Thus, following [15], we require a hierarchy of scales (see Fig. 1):

$$\lambda_0 > \lambda_L \gg \lambda_P \gg \lambda_S\,. \tag{2}$$

The local observer in a separate universe patch cannot observe the effect of ζ_L, which is locally homogeneous on the patch scale λ_P. However, today we *can* observe the effect of ζ_L on a scale λ_L within our observable universe. As we show below, by comparing different patches we will observe a modulation of the small-scale variance, due to the nonlinear coupling between ζ_L and δ_S in GR.

II. THE PHYSICAL EFFECT OF CURVATURE WITHIN THE OBSERVABLE UNIVERSE

In Newtonian gravity the only constraint on initial conditions is the Poisson equation, which provides a linear relation between the overdensity and the gravitational potential at all orders

$$\nabla^2\Phi_N = -\frac{3}{2}a^2H^2\delta\,. \tag{3}$$

Thus if the initial Newtonian potential Φ_N is Gaussian, then so is the initial density field δ. In GR, the nonlinear energy constraint equation for irrotational dust is [19]

$$\frac{2}{3}\Theta^2 - 2\sigma^2 + R^{(3)} = 16\pi G\rho + 2\Lambda\,, \tag{4}$$

where ρ is the comoving matter density, Λ is the cosmological constant, $\Theta = \nabla_\mu u^\mu$ is the expansion rate of the matter 4-velocity, σ is its shear, and $R^{(3)}$ is the Ricci curvature scalar of the 3-dimensional space orthogonal to u^μ. At first order in perturbations about an FRW cosmology, the energy constraint combines with the momentum constraint to give the relativistic version of the Poisson equation (3), where Φ_N is replaced by Φ, i.e. the spatial metric perturbation in longitudinal gauge, and δ is the synchronous comoving gauge density contrast. Note that $\Phi = 3\zeta/5$ at first order. At second order, at the start of the matter era, using the relation between $R^{(3)}$ and ζ, we obtain [6]

$$\nabla^2\zeta - 2\zeta\nabla^2\zeta + \frac{1}{2}\left(\nabla\zeta\right)^2 = -\frac{5}{2}a^2H^2\delta\,. \tag{5}$$

Consider a Gaussian distribution of ζ. We separate ζ and δ into independent long- and short-wavelength modes, $\zeta = \zeta_L + \zeta_S$ and $\delta = \delta_L + \delta_S$, where the wavelength of the long modes λ_L obeys (2); in particular, $\lambda_L \gg \lambda_P$. To leading order in ζ_S and ζ_L, and neglecting gradients of ζ_L relative to those of ζ_S, the initial constraint (5) implies $\nabla^2\zeta_L = -5a^2H^2\delta_L/2$ and

$$\nabla^2\zeta_S - 2\zeta_L\nabla^2\zeta_S = -\frac{5}{2}a^2H^2\delta_S. \tag{6}$$

The second term on the left represents the long-short mode coupling.

Within a local patch on a scale λ_P, it is possible to redefine the background spatial coordinates to absorb the effects of the long-wavelength perturbations ζ_L, following [13]:

$$\tilde{x}^i = x^i + \xi^i, \quad \xi^i = \zeta_L x^i. \tag{7}$$

If we neglect gradients of the long mode, this transformation eliminates ζ_L from the spatial metric (1)

$$ds^2_{(3)} = e^{2\zeta_L}e^{2\zeta_S}\delta_{ij}dx^idx^j = e^{2\zeta_S}\delta_{ij}d\tilde{x}^id\tilde{x}^j. \tag{8}$$

Crucially, this transformation holds only over a *single* patch (see Fig. 1). The spatial variation of ζ_L over very large scales, $\geq \lambda_L$, represents a *physical* curvature perturbation that cannot be eliminated by a coordinate transformation in a λ_P-patch.

Since this is a purely spatial coordinate transformation, the curvature and density perturbations transform as scalars,

$$\tilde{\zeta}_S(\tilde{x}) = \zeta_S(x)\,,\ \tilde{\delta}_S(\tilde{x}) = \delta_S(x)\,, \quad \text{where} \ \ \tilde{x} = \big[1+\zeta_L(x)\big]x\,. \tag{9}$$

The constraint equation (6) becomes

$$\tilde{\nabla}^2\tilde{\zeta}_S(\tilde{x}) = -\frac{5}{2}a^2H^2\tilde{\delta}_S(\tilde{x}). \tag{10}$$

Thus in the new local coordinates, in one patch of size λ_P, we have a linear Poisson equation and the long-short mode coupling appears to be absent. This confirms the fact that the local observer in a separate universe patch cannot observe the effect of the locally homogeneous perturbation ζ_L, as argued in [12, 13].

The key point, overlooked in [12, 13], is that $\tilde{\delta}_S(\tilde{x})$ *is not independent of* ζ_L. The rescaled coordinates $\tilde{x}^i$ explicitly depend on ζ_L through the transformation (7). The local variance on a comoving scale R of the small-scale density perturbation is invariant under the coordinate transformation,

$$\tilde{\sigma}^2_{\tilde{R}} = \sigma^2_R\,, \quad \text{where} \ \ \tilde{R} = (1+\zeta_L)R\,, \tag{11}$$

and is therefore modulated by the long mode ζ_L.

The original coordinates x^i define a global chart, which is essential for defining random fields such as ζ_L. The coordinates $\tilde{x}^i$ that remove curvature from a single patch are themselves *random* fields due to their dependence on ζ_L. Indeed, a coordinate transformation that depends on a random field is not a new concept in large-scale structure. The situation here is reminiscent of the redshift-space distortion map, where the random field is given by the peculiar velocities (which are in turn generated by large-scale density perturbations). Because of the nonlinear nature of this map, an initially Gaussian field in real space becomes non-Gaussian in redshift space [20, 21].

As shown in [13], $\tilde{\zeta}(x)$ and $\tilde{\delta}(x)$ are Gaussian fields in the coordinates x^i. But $\tilde{x}^i$ are also Gaussian fields in x^i – and therefore nonlinear functions of $\tilde{x}$ like $\tilde{\delta}(\tilde{x})$ are non-Gaussian. See Fig. 2 for a schematic illustration of this.

There is no argument against the fact that $\tilde{x}^i$ coordinates are useful to discuss physics in a local patch of size $\lambda_P \ll \lambda_L$. The problem is that these coordinates are useful only locally. The effect of the long mode ζ_L is to create

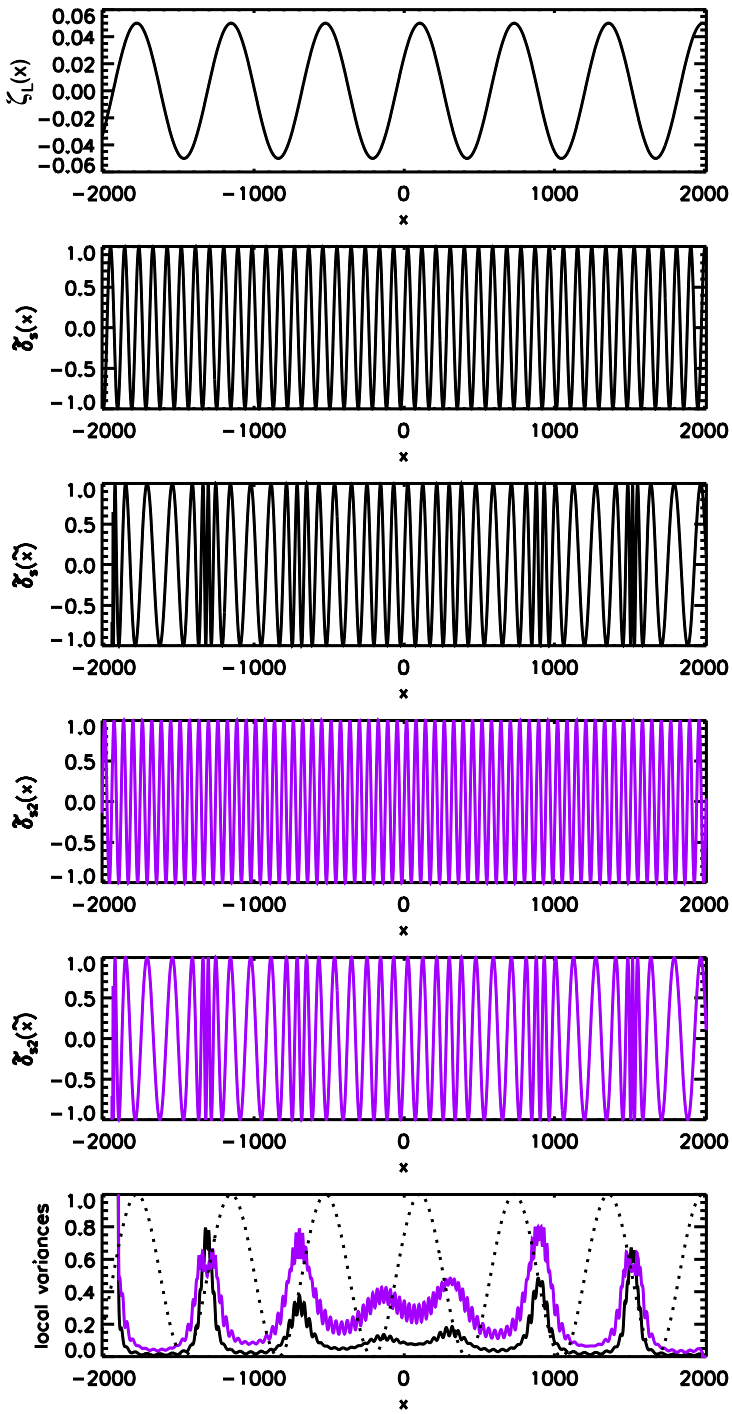

FIG. 2: Illustrating the non-Gaussianity of $\tilde{\delta}_S(\tilde{x})$ and its correlation with ζ_L. Start from the fact that $\tilde{\delta}_S(x)$ is Gaussian [13], and uncorrelated with the Gaussian $\zeta_L(x)$. The top 2 panels show one k mode for each. Then apply the transformation (9), $x \to \tilde{x} = x[1 + \zeta_L(x)]$. The resulting $\tilde{\delta}_S(\tilde{x})$ field (next panel down) is clearly modulated by ζ_L. To see that it is non-Gaussian, consider another k mode, $\tilde{\delta}_{S2}(x)$ (fourth panel). Clearly $\tilde{\delta}_{S2}(\tilde{x})$ is highly correlated to $\tilde{\delta}_S(\tilde{x})$, i.e., there are phase correlations. The local variances of these two modes are correlated with ζ_L (bottom panel).

spatial curvature $\nabla^2\zeta_L$ on a constant-time hypersurface. The curvature can be eliminated only locally, by neglecting gradients of ζ_L, as in (8). Beyond the single patch, when gradients are not negligible, we have [22]

$$ds^2_{(3)} = e^{2\zeta_L} e^{2\zeta_S} \delta_{ij} dx^i dx^j = e^{2\zeta_S}\Big[\delta_{ij} d\tilde{x}^i d\tilde{x}^j + O\big(|\tilde{x}|^2 \nabla^2\zeta_L\big)\Big] . \tag{12}$$

Indeed, all coordinate transformations that neglect curvature can only be defined locally (see [22]). The spatial curvature generated by ζ_L is directly related to the long-wavelength density perturbation δ_L by the long-wavelength part of (5). On a constant-time hypersurface, the density perturbation cannot be eliminated by the spatial coordinate transformation. This implies that we need many different local patches of scale $\sim \lambda_P$ described by different $\tilde{x}^i$-coordinates in the entire observed universe of scale $\sim \lambda_0$.

Within the patch on a scale λ_P, the local observer does not notice this stochastic nature of the local coordinates. However, once we are interested in physics beyond the local patch and we want to compare the different patches, we notice that $\tilde{x}^i$ vary stochastically through their dependence on ζ_L. This implies that $\tilde{\delta}_S(\tilde{x})$ is *not* independent of ζ_L, simply because $\tilde{x}^i$ are correlated with ζ_L. These effects can be appreciated only by looking at a region where $\nabla^2\zeta_L$ is not negligible and the stochastic nature of ζ_L becomes apparent: *curvature can only be eliminated locally.*

A more complete treatment, including explicit use of Fermi coordinates and the quantitative effects of this mode-coupling, will be presented elsewhere [23].

III. SCALE-DEPENDENT BIAS FROM SINGLE-FIELD INFLATION

In Newtonian gravity, it is standard to parametrise primordial non-Gaussianity of local type in the Newtonian potential by

$$\Phi_N = \varphi + f_{\rm NL}\left(\varphi^2 - \langle\varphi^2\rangle\right) , \tag{13}$$

where φ is a Gaussian random field. If we split the Gaussian field into long and short modes, $\varphi = \varphi_S + \varphi_L$, and use the same assumptions that lead to (6), then the Poisson equation (3) yields

$$\big(1 + 2f_{\rm NL}\varphi_L\big)\nabla^2\varphi_S = -\frac{3}{2}a^2H^2\delta_S . \tag{14}$$

Single-field, slow-roll inflation generates an almost Gaussian distribution for ζ, which remains Gaussian for adiabatic perturbations on super-Hubble scales through to the start of the matter-dominated era. For this Gaussian case, using the first-order relation $\zeta = 5\varphi/3$, the GR second-order constraint equation (6) gives

$$\Big(1 - \frac{10}{3}\varphi_L\Big)\nabla^2\varphi_S = -\frac{3}{2}a^2H^2\delta_S. \tag{15}$$

By exactly the same arguments that lead from (14) to scale-dependent galaxy bias [10, 11], it follows that (15) leads to a scale-dependent bias, with an effective (local) non-Gaussianity parameter

$$f_{\rm NL}^{\rm GR} = -\frac{5}{3} . \tag{16}$$

This is what would in principle be measured in galaxy surveys in the case of a universe with single-field slow-roll inflation [24–27]. The GR-induced long-short mode coupling is independent of inflationary effects and produces a real, physical scale-dependent bias that is measurable in principle.

While Φ_N satisfies the linear Poisson equation at all orders, ζ satisfies the nonlinear constraint (5). In GR we should define primordial non-Gaussianity, e.g., from inflation, in terms of the primordial metric perturbation, ζ. Local-type primordial non-Gaussianity can be parametrised as

$$\zeta = \frac{5}{3}\Big[\varphi + f_{\rm NL}^{\rm prim}\left(\varphi^2 - \langle\varphi^2\rangle\right)\Big]. \tag{17}$$

Thus, the parameter of local non-Gaussianity that is observable from the galaxy bias on very large scales, is in general given by

$$f_{\rm NL} = f_{\rm NL}^{\rm prim} + f_{\rm NL}^{\rm GR} = f_{\rm NL}^{\rm prim} - \frac{5}{3} . \tag{18}$$

IV. CONCLUSIONS

We have shown how non-Gaussian correlations in the matter overdensity arise due to nonlinear constraints in GR, even when the primordial metric perturbation from inflation, ζ, is described by a Gaussian random field. While the effect of the long-wavelength curvature can be removed in one separate universe patch, it cannot simultanoeusly be removed in different observed patches, leading to a curvature modulation of the short-wavelength variance. This modulation leads to the effective $f_{\rm NL}^{\rm GR} = -5/3$ in (16) from GR effects in the matter era. It may be simply understood as arising from the long-wavelength metric perturbation ζ_L rescaling the local small-scale curvature perturbation ζ_S, and thus the local small-scale density field through the nonlinear constraint equation (6).

If the long-wavelength perturbation were much larger than our observable horizon, $\lambda_L \gg \lambda_0$, then that would be the end of the story – very long-wavelength perturbations much larger than our horizon form part of our background cosmology and cannot be observed locally. However, the fact that one can absorb the effect of a long-wavelength mode into the background geometry *locally* does not mean that it is unobservable if the long-wavelength mode varies within our observable horizon, $\lambda_0 > \lambda_L \gg \lambda_P$. This variation is evident in (6), which shows the explicit dependence of $\delta_S(x)$ on ζ_L, but it is still present when the small-scale perturbations are expressed in terms of the rescaled coordinates (7). In particular, the local coordinate rescaling (7) is itself a function of the random field ζ_L.

The density is a scalar quantity on a constant-time hypersurface under the spatial coordinate transformation, as in (9). Physics is not changed by this coordinate transformation and the local variance is also invariant, $\sigma^2_R = \tilde\sigma^2_{\tilde R}$. In x coordinates, the relation between the density and the curvature perturbation is nonlinear, which creates the long-short mode coupling for $\delta_S(x)$ as in (6). In $\tilde x$ coordinates, the relation between the density and curvature is linear as in (10) – but the coordinates $\tilde x$ are modulated by the long mode, which creates the *same long-short mode coupling* for $\tilde\delta_S(\tilde x)$. For example, the three-point function is

$$\left\langle \tilde\zeta_L(\tilde x_1)\tilde\delta_S(\tilde x_2)\tilde\delta_S(\tilde x_3)\right\rangle = \left\langle \zeta_L(x_1)\delta_S(x_2)\delta_S(x_3)\right\rangle \neq 0. \tag{19}$$

Thus the small-scale perturbation expressed in $\tilde x$ coordinates, $\tilde\delta_S(\tilde x)$, is still correlated with ζ_L. Note that the long/small-wavelength split recovers the so-called squeezed limit of the bispectrum. The GR contribution to the bispectrum has a specific shape dependence away from the squeezed limit (see Eq. 12 of [1] or Eq. 14 of [2]). These points will be further discussed in [23].

This recalls similar issues already discussed in relation to non-Gaussianities in the CMB that arise a recombination due solely to the intrinsic nonlinearity of relativistic perturbations [28–30]. If the long-wavelegth perturbations are on scales greater than the present horizon, they cannot have any physical effect, and can be rescaled away (this corresponds to properly redefining the background average temperature [31, 32]). However, we are interested in modes that are inside the horizon today, and we need to compare different patches of the sky modulated by a long mode [31, 33]. This provides an alternative understanding of a GR term that was missing in [30], compared to the expression for the squeezed CMB angular bispectrum obtained in [29, 31, 34].

The effect of an adiabatic perturbation ζ_L, of sufficiently long wavelength λ_L, can be understood locally (in a patch of size $\lambda_P \ll \lambda_L$) as perturbing the background geometry and density. This allows one to derive useful consistency conditions for nonlinear correlations between long- and short-wavelength modes, corresponding to n-point functions in certain squeezed limits [35–40]. Indeed this is exactly the origin of the Maldacena relation [41, 42] between the squeezed limit of the bispectrum for primordial metric perturbations produced during single-field inflation and the scale-dependence of the power spectrum. The angular bispectrum of CMB temperature anisotropies (and polarization) has been estimated [29, 31] and shown to reproduce in this squeezed limit the full numerical calculations of the Einstein-Boltzmann system at second order [43–46]. This intrinsic non-Gaussianity in the CMB, predicted by local rescaling arguments, could in principle be observed by future experiments.

The intrinsic nonlinearity of GR constraint equations imposes non-zero long-short mode-coupling in the large-scale density field, even in the absence of any primordial non-Gaussianity from inflation. Despite recent claims, this effect is real and physical – and it leaves potentially observable signatures in the large-scale strucure of the Universe, providing a robust and important target for future observational probes.

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[1] N. Bartolo, S. Matarrese and A. Riotto, "Signatures of primordial non-Gaussianity in the large-scale structure of the Universe," JCAP **0510**, 010 (2005) [astro-ph/0501614].

[2] L. Verde and S. Matarrese, "Detectability of the effect of Inflationary non-Gaussianity on halo bias," Astrophys. J. **706**, L91 (2009) [arXiv:0909.3224].

[3] N. Bartolo, S. Matarrese, O. Pantano and A. Riotto, "Second-order matter perturbations in a ΛCDM cosmology and non-Gaussianity," Class. Quant. Grav. **27**, 124009 (2010) [arXiv:1002.3759].

[4] A. L. Fitzpatrick, L. Senatore and M. Zaldarriaga, "Contributions to the Dark Matter 3-Pt Function from the Radiation Era," JCAP **1005**, 004 (2010) [arXiv:0902.2814].

[5] M. Bruni, J. C. Hidalgo, N. Meures and D. Wands, "Non-Gaussian Initial Conditions in ΛCDM: Newtonian, Relativistic, and Primordial Contributions," Astrophys. J. **785**, 2 (2014) [arXiv:1307.1478].

[6] M. Bruni, J. C. Hidalgo and D. Wands, "Einstein's signature in cosmological large-scale structure," Astrophys. J. **794**, L11 (2014) [arXiv:1405.7006].

[7] C. Uggla and J. Wainwright, "Simple expressions for second order density perturbations in standard cosmology," Class. Quant. Grav. **31**, 105008 (2014) [arXiv:1312.1929].

[8] E. Villa, L. Verde and S. Matarrese, "General relativistic corrections and non-Gaussianity in large-scale structure," Class. Quantum Grav. **31**, 234005 (2014) [arXiv:1409.4738].

[9] E. Villa and C. Rampf, "Relativistic perturbations in ΛCDM: Eulerian and Lagrangian approaches," arXiv:1505.04782.

[10] N. Dalal, O. Doré, D. Huterer and A. Shirokov, "The imprints of primordial non-gaussianities on large-scale structure: scale dependent bias and abundance of virialized objects," Phys. Rev. D **77**, 123514 (2008) [arXiv:0710.4560].

[11] S. Matarrese and L. Verde, "The effect of primordial non-Gaussianity on halo bias," Astrophys. J. **677**, L77 (2008) [arXiv:0801.4826].

[12] L. Dai, E. Pajer and F. Schmidt, "On Separate Universes," arXiv:1504.00351.

[13] R. de Putter, O. Doré and D. Green, "Is There Scale-Dependent Bias in Single-Field Inflation?," arXiv:1504.05935.

[14] D. S. Salopek and J. R. Bond, "Nonlinear evolution of long wavelength metric fluctuations in inflationary models," Phys. Rev. D **42**, 3936 (1990).

[15] D. Wands, K. A. Malik, D. H. Lyth and A. R. Liddle, "A New approach to the evolution of cosmological perturbations on large scales," Phys. Rev. D **62**, 043527 (2000) [astro-ph/0003278].

[16] D. H. Lyth and Y. Rodriguez, "The Inflationary prediction for primordial non-Gaussianity," Phys. Rev. Lett. **95**, 121302 (2005) [astro-ph/0504045].

[17] D. H. Lyth, K. A. Malik and M. Sasaki, "A General proof of the conservation of the curvature perturbation," JCAP **0505**, 004 (2005) [astro-ph/0411220].

[18] D. Langlois and F. Vernizzi, "Evolution of nonlinear cosmological perturbations," Phys. Rev. Lett. **95**, 091303 (2005) [astro-ph/0503416].

[19] G. F. R. Ellis, R. Maartens and M. A. H. MacCallum, *Relativistic Cosmology*, Cambridge University Press (2012).

[20] R. Scoccimarro, "Redshift-space distortions, pairwise velocities and nonlinearities," Phys. Rev. D **70**, 083007 (2004) [astro-ph/0407214].

[21] J. R. Shaw and A. Lewis, "Non-linear Redshift-Space Power Spectra," Phys. Rev. D **78**, 103512 (2008) [arXiv:0808.1724].

[22] F. K. Manasse and C. W. Misner, "Fermi Normal Coordinates and Some Basic Concepts in Differential Geometry," J. Math. Phys. **4**, 735 (1963).

[23] N. Bartolo et al., in preparation.

[24] C. Carbone, L. Verde and S. Matarrese, "Non-Gaussian halo bias and future galaxy surveys," Astrophys. J. **684**, L1 (2008) [arXiv:0806.1950].

[25] C. Carbone, O. Mena and L. Verde, "Cosmological Parameters Degeneracies and Non-Gaussian Halo Bias," JCAP **1007**, 020 (2010) [arXiv:1003.0456].

[26] S. Camera, M. G. Santos and R. Maartens, "Probing primordial non-Gaussianity with SKA galaxy redshift surveys: a fully relativistic analysis," Mon. Not. Roy. Astron. Soc. **448**, 1035 (2015) [arXiv:1409.8286].

[27] S. Camera, R. Maartens and M. G. Santos, "Einstein's legacy in galaxy surveys," Mon. Not. Roy. Astron. Soc. Lett., in press (2015) [arXiv:1412.4781].

[28] S. Mollerach and S. Matarrese, "Cosmic microwave background anisotropies from second order gravitational perturbations," Phys. Rev. D **56**, 4494 (1997) [astro-ph/9702234].

[29] N. Bartolo, S. Matarrese and A. Riotto, "Gauge-invariant temperature anisotropies and primordial non-Gaussianity," Phys. Rev. Lett. **93**, 231301 (2004) [astro-ph/0407505].

[30] P. Creminelli and M. Zaldarriaga, "CMB 3-point functions generated by nonlinearities at recombination," Phys. Rev. D **70**, 083532 (2004) [astro-ph/0405428].

[31] P. Creminelli, C. Pitrou and F. Vernizzi, "The CMB bispectrum in the squeezed limit," JCAP **1111**, 025 (2011) [arXiv:1109.1822].

[32] L. Boubekeur, P. Creminelli, G. D'Amico, J. Norena and F. Vernizzi, "Sachs-Wolfe at second order: the CMB bispectrum on large angular scales," JCAP **0908**, 029 (2009) [arXiv:0906.0980].

[33] M. Mirbabayi and M. Zaldarriaga, "CMB Anisotropies from a Gradient Mode," JCAP **1503**, 056 (2015) [arXiv:1409.4777].

[34] N. Bartolo, S. Matarrese and A. Riotto, "Non-Gaussianity in the Cosmic Microwave Background Anisotropies at Recombination in the Squeezed limit," JCAP **1202**, 017 (2012) [arXiv:1109.2043].

[35] A. Kehagias and A. Riotto, "Symmetries and Consistency Relations in the Large Scale Structure of the Universe," Nucl. Phys. B **873**, 514 (2013) [arXiv:1302.0130].

[36] P. Creminelli, J. Norena, M. Simonovic and F. Vernizzi, "Single-Field Consistency Relations of Large Scale Structure," JCAP **1312**, 025 (2013) [arXiv:1309.3557].

[37] M. Peloso and M. Pietroni, "Ward identities and consistency relations for the large-scale structure with multiple species," JCAP **1404**, 011 (2014) [arXiv:1310.7915].

[38] P. Creminelli, J. Gleyzes, M. Simonovic and F. Vernizzi, "Single-Field Consistency Relations of Large Scale Structure. Part II: Resummation and Redshift Space," JCAP **1402**, 051 (2014) [arXiv:1311.0290].

[39] P. Creminelli, J. Gleyzes, L. Hui, M. Simonovic and F. Vernizzi, "Single-Field Consistency Relations of Large Scale Structure. Part III: Test of the Equivalence Principle," JCAP **1406**, 009 (2014) [arXiv:1312.6074].

[40] A. Kehagias, A. M. Dizgah, J. Norena, H. Perrier and A. Riotto, "A Consistency Relation for the Observed Galaxy Bispectrum and the Local non-Gaussianity from Relativistic Corrections," arXiv:1503.04467.

[41] J. M. Maldacena, "Non-Gaussian features of primordial fluctuations in single field inflationary models," JHEP **0305**, 013 (2003) [astro-ph/0210603].

[42] P. Creminelli and M. Zaldarriaga, "Single field consistency relation for the 3-point function," JCAP **0410**, 006 (2004) [astro-ph/0407059].

[43] Z. Huang and F. Vernizzi, "Cosmic Microwave Background Bispectrum from Recombination," Phys. Rev. Lett. **110**, 101303 (2013) [arXiv:1212.3573].

[44] S.-C. Su, E. A. Lim and E. P. S. Shellard, "Cosmic microwave background bispectrum from nonlinear effects during recombination," Phys. Rev. D **90**, 023004 (2014) [arXiv:1212.6968].

[45] G. W. Pettinari, C. Fidler, R. Crittenden, K. Koyama and D. Wands, "The intrinsic bispectrum of the Cosmic Microwave Background," JCAP **1304**, 003 (2013) [arXiv:1302.0832].

[46] G. W. Pettinari, C. Fidler, R. Crittenden, K. Koyama, A. Lewis and D. Wands, "Impact of polarization on the intrinsic cosmic microwave background bispectrum," Phys. Rev. D **90**, 103010 (2014) [arXiv:1406.2981].